Fully distributed adaptive consensus protocol with bounded gains

Štefan Knotek

Dept. of Control Engineering, Czech Technical University, Karlovo náměstí 13, 121 35 Praha, Czech Republic

stefan.knotek@fel.cvut.cz

Abstract. *This paper introduces a novel distributed adaptive consensus protocol to solve the distributed consensus problem for multi-agent systems with general linear time-invariant dynamics and undirected connected communication graphs. The presented protocol proposes a solution to the problems of recent adaptive consensus protocols with large or unbounded coupling gains by introducing a novel coupling gain dynamics, that allows the coupling gains to synchronize and decay to some estimated value. The proposed protocol does not require any centralized information, therefore it can be implemented on agents in fully distributed fashion.*

Keywords

distributed control, adaptive control, multi-agent systems, cooperative regulator problem, relative-state consensus

1. Introduction

In last two decades, a great effort has been made in distributed control and estimation in formation of mobile robots, satellites and vehicles. Previously developed theoretical results in control of single-agent system motivated the designs of recent distributed cooperative controllers and observers using state or output-feedback in continuous and discrete-time. The static consensus protocols presented in [1], [4] and [5] use a state-feedback with coupling gain that satisfies a bound calculated from the smallest non-zero real part of Laplacian eigenvalues. The graph structure has to be known to calculate this bound. Therefore centralized information is required by each agent.

Distributed adaptive consensus protocols propose a solution to this problem on undirected connected graph [2] as well as on directed graphs having a spanning tree with leader as a root node [3]. These protocols do not rely on any centralized information, therefore they can be implemented by each agent separately without using any global information. The protocols guarantee cooperative stability, however benefits from adaptability suffers from possibly large control effort and lack of robustness to noise.

In this paper we introduce an adaptive consensus protocol on undirected connected graphs consisting of identical agents with linear time-invariant (LTI) dynamics. The protocol introduces a novel coupling gain dynamics allowing the coupling gains to synchronize and decay. This is found to solve the above mentioned difficulties of recently proposed adaptive consensus protocols.

This paper is organized as follows. Section 2 introduces the basic notation and graph preliminaries used throughout the paper. Section 3 states the problems of recent adaptive consensus protocols. The novel adaptive consensus protocol is presented in Section 4. Numerical simulation verifying the introduced protocol are given in Section 6. Section 7 concludes the paper.

2. Preliminaries

In this paper the following notations and definitions are used. $\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ real matrices. A matrix $M = \mathrm{diag}(v)$ for $v \in \mathbb{R}^n$ denotes $\mathbb{R}^{n\times n}$ diagonal matrix with elements of v on the diagonal. $I_N \in \mathbb{R}^{N\times N}$ is the identity matrix. Positive (semi)-definite symmetric matrix is denoted by $M \succ (\succeq)0$. Positive (non-negative) vector denoted by $v > (\geq)0$ has all positive (non-negative) entries. The sum over all agents is denoted by $\sum_i$ for $i = 1, \ldots, N$.

An undirected graph is given by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a non-empty finite set of vertices and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of edges. An edge is a pair of nodes (v_i, v_j), $v_i \neq v_j$, representing that agents i and j can exchange information between them. In sequel, the graph $\mathcal{G}$ is assumed to be undirected, connected and simple.

The adjacency matrix $E = [e_{ij}] \in \mathbb{R}^{N\times N}$ associated with the graph $\mathcal{G}$ is defined by $e_{ij} = e_{ji} > 0$ if $(v_i, v_j) \in \mathcal{E}$, otherwise $e_{ij} = e_{ji} = 0$. Define the vector of node degrees as $d = E\mathbf{1}_N$, and the degree matrix as $D = \mathrm{diag}(d)$. Then the graph Laplacian is defined by $L = D - E$.

3. Problem statement

Consider a graph $\mathcal{G}$ consisting of N identical agents with general LTI dynamics

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^m$ is the input, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. The matrix A is not necessarily stable but the double of matrices (A, B) is assumed to be stabilizable.

The goal is to design a control law to solve the cooperative regulator problem in sense of $\lim_{t\to\infty} ||x_j - x_i|| = 0, \forall i, j$ without requiring any centralized information. A solution to this problem is proposed in [2] by an adaptive consensus protocol

$$u_i = K \textstyle\sum_j c_{ij} e_{ij}(x_i - x_j), \quad i = 1, \ldots, N \tag{2}$$

$$\dot{c}_{ij} = \chi_{ij} e_{ij}(x_i - x_j)^T \Gamma (x_i - x_j) \tag{3}$$

where $\chi_{ij} = \chi_{ji}$ are positive constants, c_{ij} is the time-varying coupling gain between i-th and j-th agent and $K \in \mathbb{R}^{m\times n}$ and $\Gamma \in \mathbb{R}^{n\times n}$ are the feedback and adaptation gain matrices, respectively.

The recently introduced adaptive consensus protocols [2], [3] do not require any centralized information, therefore the agents can be implemented in a fully distributed fashion. However, they also introduce several drawbacks.

Since the coupling gains' derivative is a monotonically increasing function, the coupling gain values rise as long as there is some error $||x_i - x_j|| \neq 0$ for any i, j. Hence the farther the initial conditions of the agents are, the higher the final values of the coupling gains. The coupling gains might therefore attain higher value than it is necessary for the network stability. They are also decoupled, therefore in general they end up with different final values and the network gets unbalanced.

Additionally, if there is any noise in state measurements, the coupling gains would rise reaching some saturation level. Therefore, instead of implementing the adaptive consensus protocol, the coupling gains could from the outset be initialized to this saturation value.

Thus, to avoid these difficulties we introduce a novel adaptive consensus protocol that allows the coupling gains to synchronize and decay.

4. Adaptive consensus protocol

Let each agent implement a control input in the form

$$u_i = c_i K \textstyle\sum_j e_{ij}(x_j - x_i), \quad i = 1, \ldots, N \tag{4}$$

where c_i is the time-varying coupling gain associated with i-th agent. An i-th agent dynamics is then given by

$$\dot{x}_i = Ax_i + c_i BK \textstyle\sum_j e_{ij}(x_j - x_i) \tag{5}$$

Let each agent implement the coupling gain dynamics

$$\dot{c}_i = \textstyle\sum_j e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i) + \sum_j e_{ij}(c_j - c_i) - \ell(c_i - \kappa_i) \tag{6}$$

where $\ell > 0$ is a constant, $\kappa_i \geq 0$ is a constant estimated by an i-th agent and $\Gamma \in \mathbb{R}^{n\times n}$ is the adaptation gain matrix.

The gain matrices K and Γ are designed by LQR method. Let $Q = Q^T \in \mathbb{R}^{n\times n}$ and $R = R^T \in \mathbb{R}^{m\times m}$ be positive definite symmetric matrices, then

$$K = R^{-1} B^T P \tag{7}$$

$$\Gamma = PBK \tag{8}$$

where positive definite matrix $P \in \mathbb{R}^{n\times m}$ is the unique solution of the algebraic Riccati equation

$$0 = A^T P + PA + Q - PBR^{-1} B^T P. \tag{9}$$

The introduced adaptive consensus protocol (4, 6) is motivated by [2], however there are several major differences. Note that different from [2], each agent has only one adaptive gain, much along the line of [3], that is required for the coupling gain synchronization.

The coupling gain dynamics (6) is not a monotonically increasing function. It consists of three main terms. The first term on the right-hand side is the non-negative quadratic term motivated by [2]. Its purpose is to push the coupling gains to higher values until the states get synchronized. The second term on the right-hand side synchronizes the coupling gains and thereby solves the above mentioned problem with different coupling gains. The third term on the right-hand side pushes the coupling gains to κ_i and by this solves the problem with high gains. The value of κ_i is estimated by an estimation algorithm. The strength of the decay term is determined by the constant ℓ.

Assume the coupling gain dynamics (6) with $\kappa_i = 0$, $\forall i$, then the network dynamic is give by

$$\dot{x}_i = Ax_i + c_i BK \textstyle\sum_j e_{ij}(x_j - x_i) \tag{10}$$

$$\dot{c}_i = \textstyle\sum_j e_{ij}(x_j - x_i)^T \Gamma (x_j - x_i) + \sum_j e_{ij}(c_j - c_i) - \ell c_i. \tag{11}$$

Define the virtual leader $x_0 = \frac{1}{N}\sum_i x_i$, the virtual tracking error $\delta_i = x_i - x_0$ and the coupling gain transformation $z_i = c_i - \beta$, where β is some positive constant. Then the the network dynamics (10, 11) transformed to the new coordinates (δ_i, z_i) is given by

$$\dot{\delta}_i = A\delta_i + z_i BK \textstyle\sum_j e_{ij}(\delta_j - \delta_i) + \beta BK \sum_j e_{ij}(\delta_j - \delta_i) \tag{12}$$

$$\dot{z}_i = \textstyle\sum_j e_{ij}(\delta_j - \delta_i)^T \Gamma (\delta_j - \delta_i) + \sum_j e_{ij}(z_j - z_i) - \ell z_i - \ell\beta. \tag{13}$$

This network dynamics consists of some nominal dynamic and the non-vanishing perturbation term $(-\ell\beta)$, therefore note it as the perturbed nominal dynamics. If the nominal dynamics is globally exponentially stable, following [6, Lem. 5.2] it can be shown, that the perturbed nominal dynamics (12, 13) is uniformly ultimately bounded.

Lemma 1. *Consider the network dynamics (5, 6) with $\kappa_i = 0$, $\forall i$. Then its solution is globally uniformly ultimately bounded, i.e. there exist a positive constant μ and ν, and for every $\epsilon \in (0, \nu)$ there is a positive constant $t_\mu = t_\mu(\epsilon)$ such that if $\|\delta(t_0), z(t_0)\| \leq \epsilon$, then $\|\delta(t), z(t)\| < \mu$, $\forall t \geq t_0 + t_\mu$.*

Since the proof of Lemma 1 is extensive it will be published later. Lemma 1 reveals an important property of the network dynamics (5, 6). It says that for $\kappa_i = 0$, $\forall i$, the network dynamics is uniformly ultimately bounded, i.e. in the worst case the solution of the network dynamics ends up in a bounded set.

Having uniform ultimate boundedness is necessary, but not sufficient for practical implementation of the protocol. For that reason we introduce the full form of the proposed coupling gain dynamics (6).

In comparison to the coupling gain dynamics from Lemma 1, it contains an additional term ($+l\kappa_i$), that can cancel effects of the non-vanishing perturbation term ($-l\beta$).

Assuming coupling gain dynamics (6), the total network error dynamics reads

$$\dot{\delta}_i = A\delta_i + z_i BK \sum_j e_{ij}(\delta_j - \delta_i) + \beta BK \sum_j e_{ij}(\delta_j - \delta_i) \tag{14}$$

$$\begin{aligned} \dot{z}_i = -\ell z_i + \ell\kappa_i - \ell\beta + \sum_j e_{ij}(z_j - z_i) \\ + \sum_j e_{ij}(\delta_j - \delta_i)^T \Gamma(\delta_j - \delta_i). \end{aligned} \tag{15}$$

By increasing each κ_i, effects of non-vanishing perturbation term are gradually reduced until the condition $\kappa_i = \beta$, $\forall i$ is met. If $\kappa_i \geq \beta$, $\forall i$ the non-vanishing perturbation term is cancelled, the dynamics reduces to the case of nominal dynamics and it gets globally exponentially stable.

5. Estimation algorithm

The uniform ultimate boundedness provides time necessary to estimate the value of κ_i and get the network dynamics to exponentially stable region. The goal is now to estimate κ_i from the i-th agent trajectory. We require low value of κ_i to minimize the control effort, but at the same time sufficiently high to cancel effects of the non-vanishing perturbation.

The proposed estimation algorithm is based on the interval-halving method. Presence of the non-vanishing perturbation in the network dynamics is recognized from oscillating trajectory. Note that oscillating trajectory implies oscillating coupling gain. Each agent samples the coupling gain value c_i by the sampling frequency f_s and records it in the time window $\triangle t$. The higher and the lower recorded values are averaged and substituted for κ_i. This procedure is continuously repeated. When the oscillations and thus also the perturbation vanish, the network dynamics reaches the global exponential stability.

Note that the sampling frequency f_s has to be chosen according to the Nyquist-Shannon sampling theorem. The sampling rate must exceed $2f_{max}$ and the time window $\triangle t$ must be greater than $1/f_{min}$, where f_{max} and f_{min} are the highest and the lowest frequency in the system. The decay-rate, size of non-vanishing perturbation and thus also the frequency and amplitude of oscillations are determined by the positive constant ℓ.

6. Simulation results

The adaptive control protocol (4, 6) has been simulated on a graph $\mathcal{G}$ consisting of agents described by linear double integrator dynamics

$$\dot{x_i} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i, \quad x_i = \begin{bmatrix} x_{i_1} \\ x_{i_2} \end{bmatrix}, \quad \forall i. \tag{16}$$

The interval-halving algorithm for estimation of κ_i uses the time window $\triangle t = 5s$ and the sampling frequency $f_s =$ 10Hz. Initial conditions of the agents are

$$x_{i_1}(0) \in \langle -10, 10 \rangle, \quad x_{i_2}(0) = 0, \quad c_i(0) = 0, \quad \forall i. \tag{17}$$

Each figure shows two simulations. The simulations of proposed adaptive control protocol (4, 6) with the algorithm for estimation of κ_i are situated on the top of the figures. The simulations of existing adaptive consensus protocol [2] are situated on the bottom. In Figures, δ denotes the vector of errors in state x_{i_1} and c denotes the vector of coupling gains.

The simulations on the circular graph consisting of 50 agents are shown in Figure 1. The proposed protocol reaches lower coupling gains with preserving stability.

Assuming small noise acting on states, the responses of 10 agents in circular topology are shown in Figure 2. From the figure it follows that the proposed protocol is robust to noise acting on states.

Figure 3 shows the response to the change in the network topology. At the time instance of 30 seconds the graph topology was switched from the circular graph of 4 synchronized agents to the path graph and 5-th agent was connected to the end of path. The distance of 5-th agent from the rest of the network was chosen to be 10. The proposed protocol reaches lower values of coupling gains than the existing adaptive consensus protocol [2], therefore it is found robust to the change of the graph topology.

7. Conclusion

In this paper a novel distributed adaptive consensus protocol is proposed, offering solution to the encountered problems with high gains of recently proposed adaptive con-

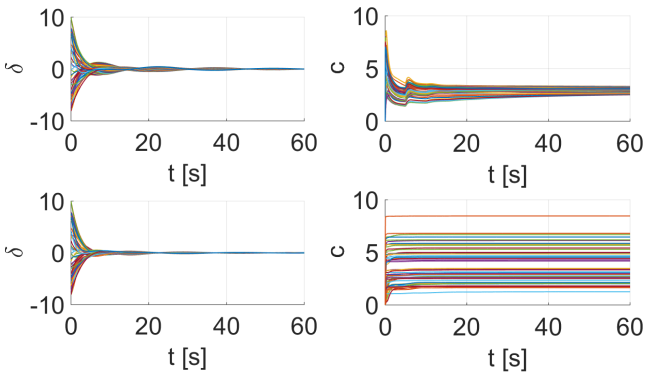

Fig. 1: Simulation of 50 agents on a circular topology.

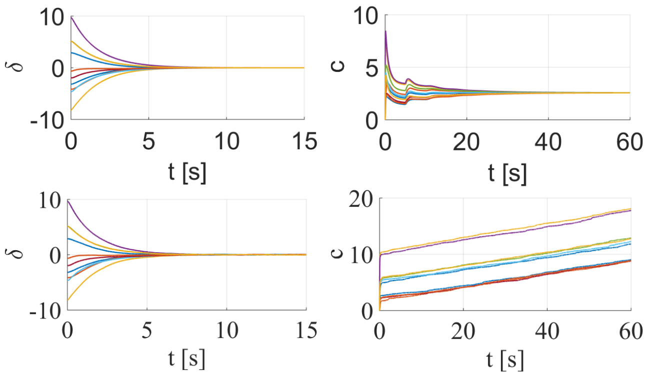

Fig. 2: Simulation of 10 agents on a circular topology assuming noise in state measurements.

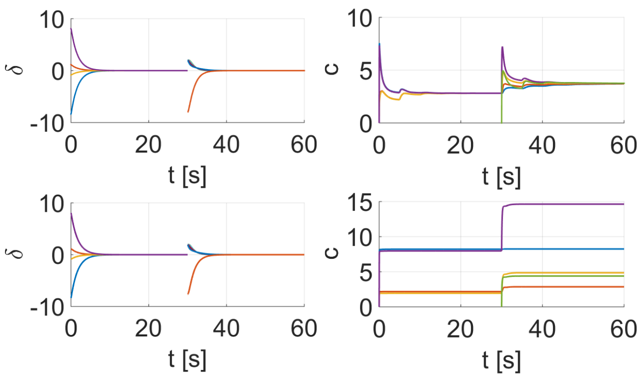

Fig. 3: Simulation of 5 agents by change in a graph topology.

sensus protocols. A coupling gain dynamics is introduced, allowing the coupling gains to synchronize and decay to some estimated value κ_i. For estimation of κ_i, an estimation algorithm based on the interval-halving method is introduced.

A comparison of the introduced protocol with the existing one is shown by simulations. The introduced protocol preserves stability with lower coupling gain values, lower control effort and robustness to the measurement noise acting on states.

The proper choice of the decay rate ℓ and the design of algorithms for estimation of κ_i is a topic of future research.

Acknowledgements

Research described in the paper was supervised by Prof. Michael Šebek and Kristian Hengster-Movric, Ph.D., FEL CTU in Prague and supported by the Grant Agency of the Czech Technical University in Prague, under grant No. SGS16/232/OHK3/3T/13.

References

[1] LI, Z., DUAN, Z., CHEN, G., HUANG, L. Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2010, vol. 57, no. 1, p. 213 - 224.

[2] LI, Z., REN, W., LIU, X., FU, M. Consensus of Multi-Agent Systems With General Linear and Lipschitz Nonlinear Dynamics Using Distributed Adaptive Protocols. *IEEE Transactions on Automatic Control*, 2013, vol. 58, no. 7, p. 1786 - 1791.

[3] LI, Z., WEN, G., DUAN, Z., REN, W. Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs. *IEEE Transactions on Automatic Control*, 2015, vol. 60, no. 4, p. 1152 - 1157.

[4] ZHANG, H., LEWIS, F. L., DAS, A. Optimal Design for Synchronization of Cooperative Systems: State Feedback, Observer and Output Feedback. *IEEE Transactions on Automatic Control*, 2011, vol. 56, no. 8, p. 1948 - 1952.

[5] ZHANG, H., LEWIS, F. L., QU, Z. Lyapunov, Adaptive, and Optimal Design Techniques for Cooperative Systems on Directed Communication Graphs. *IEEE Transactions on Industrial Electronics*, 2012, vol. 59, no. 7, p. 3026 - 3041.

[6] KHALIL, H. K.*Nonlinear Systems*. 2nd ed. Prentice-Hall, Upper Saddle River, NJ, 1996.

About Authors...

Štefan KNOTEK received the B.S. and M.S. degrees in industrial informatics and robotics from Slovak University of Technology in Bratislava, Slovak Republic, in 2012 and 2014, respectively. He is currently a Ph.D. student at the Department of Control Engineering, FEL, CTU in Prague. His area of interest includes distributed control and multi-agent systems.